Name solutions

September 22, 2010

ECE 311 Exam 1 Fall 2010

Closed Text and Notes

1) Be sure you have 12 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) The last 2 pages contain equations that may be of use to you.

6) You can leave π and ε_0 in your answers.

7) This exam is worth 100 points.

(6 pts) 1. Express the point $(3, \frac{\pi}{4}, 3)$ in Cartesian and spherical coordinates.

Cartesian:

$x = \rho \cos\phi = 3 \cos\frac{\pi}{4} = 3\frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$y = \rho \sin\phi = 3 \cos\frac{\pi}{4} = 3\frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, 3\right)$

spherical:

$r = \sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2} = \sqrt{(3)^2+(3)^2} = \sqrt{18} = 3\sqrt{2}$

$\cos\theta = \frac{z}{r} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

$\left(3\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{4}\right)$

(5 pts) 2. In cylindrical coordinates, a unit normal vector to the plane $\phi = 45°$ is

A) $\mathbf{a}_\rho$

[✓ B) $\mathbf{a}_\phi$]

C) $\mathbf{a}_z$

D) none of the above

(5 pts) 3. The intersection of the surfaces $\rho = 1$m and $z = \frac{\pi}{3}$ is

a) a sphere

[✓ b) a circle]

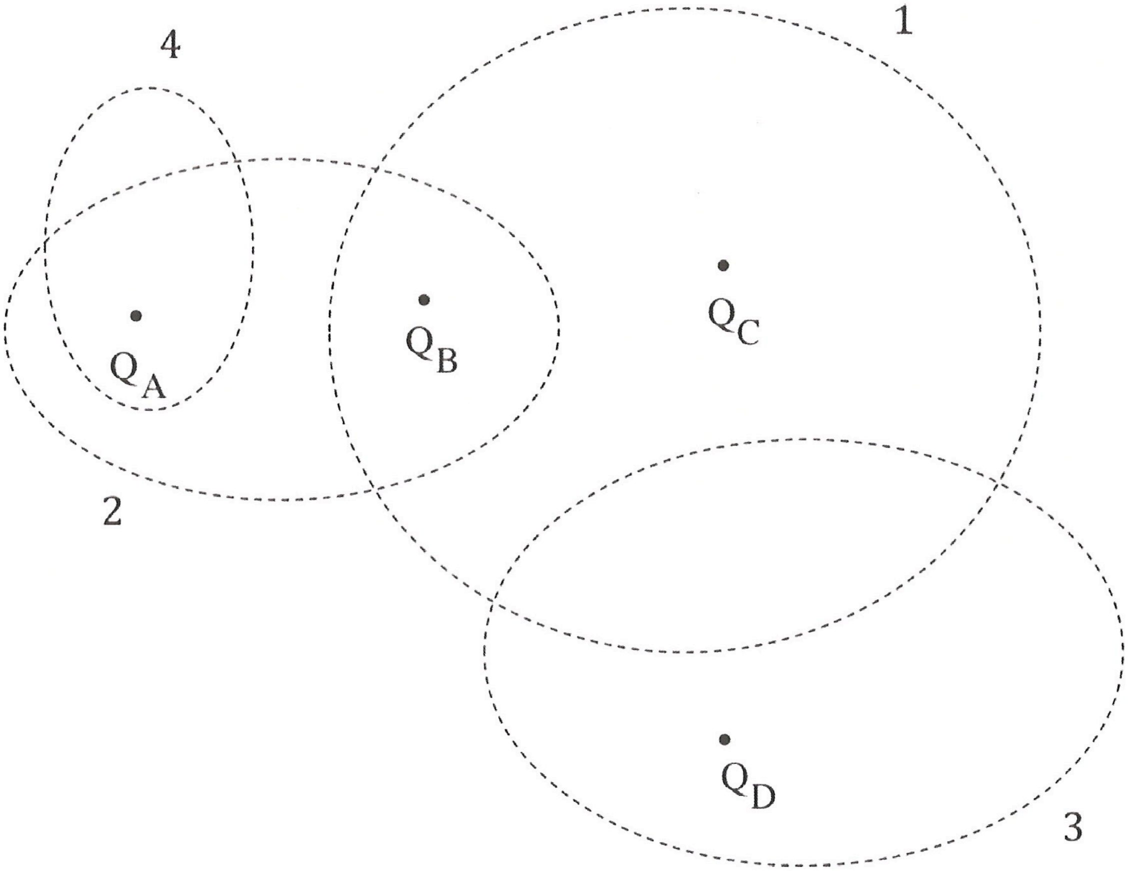
c) a straight line

d) a cone

e) c and d

f) a and b

(8 pts) 4. In the following figure the dashed lines represent closed spherical surfaces that completely surround the point charges shown, Q_A, Q_B, Q_C, and Q_D.



Determine values for these four point charges that would make,

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 1 = 2 C $\Rightarrow Q_B + Q_C = 2C \Rightarrow Q_C = 2C - Q_B = 3C$

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 2 = 0 $\Rightarrow Q_A + Q_B = 0 \Rightarrow Q_B = -Q_A = -1C$

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 3 = -1 C $\Rightarrow Q_D = -1C$

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 4 = 1 C $\Rightarrow Q_A = 1C$

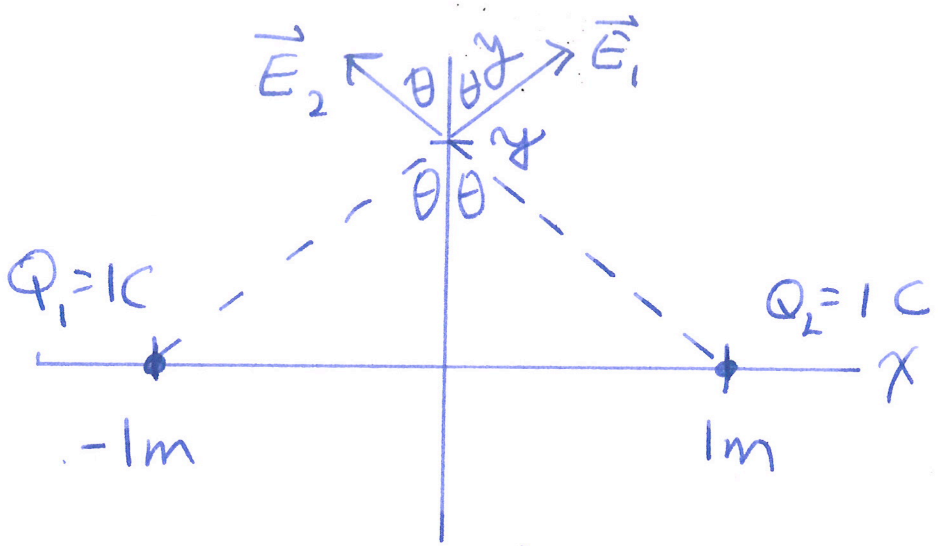
Q_A = 1C

Q_B = -1C

Q_C, = 3C

Q_D = -1C

(15 pts) 5. A 1C charge is placed at (1m, 0, 0) and a 1C charge is placed at (-1m, 0, 0). Determine the electric field intensity for the (0, y, 0), the y-axis



$\vec{E}(0, y, 0) = \vec{E}_1(0, y, 0) + \vec{E}_2(0, y, 0)$

$\vec{E}_{2x}(0, y, 0) = -\vec{E}_{1x}(0, y, 0)$

so there will only be a y-component

$\vec{E}_{2y}(0, y, 0) = \vec{E}_{1y}(0, y, 0) = E_1 \cos\theta \ (\hat{a}_y)$

$\vec{E}(0, y, 0) = 2 E_{1y}(0, y, 0) = 2 E_1(0, y, 0) \cos\theta \ \hat{a}_y$

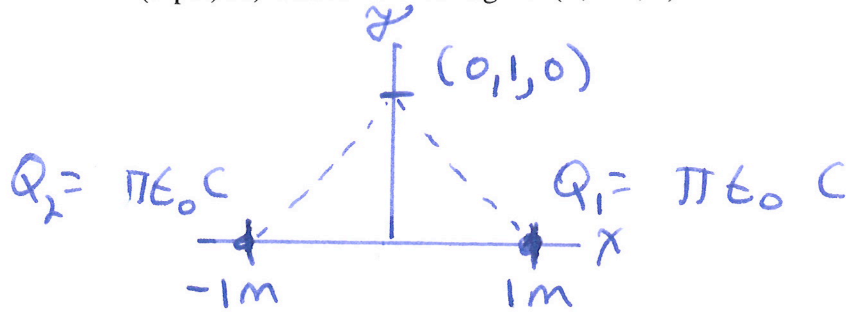
$$= 2 \frac{1}{4\pi\epsilon_0} \frac{1}{(1+y^2)} \frac{y}{(1+y^2)^{1/2}} \hat{a}_y$$

$$\vec{E}(0, y, 0) = \frac{y}{2\pi\epsilon_0 (1+y^2)^{3/2}} \hat{a}_y$$

(10 pts) 6. A $Q_1 = \pi\varepsilon_O$ C charge is placed at (1m, 0, 0) and a $Q_2 = \pi\varepsilon_O$ C charge is placed at (-1m, 0, 0).

With $V(\infty) = 0$,

(5 pts) A) what is the voltage at (0, 1m, 0)



$$V(0,1m,0) = \frac{1}{4\pi\epsilon_0}\frac{\pi\epsilon_0}{\sqrt{1+1}} + \frac{1}{4\pi\epsilon_0}\frac{\pi\epsilon_0}{\sqrt{1+1}}$$

$$= \frac{1}{2}\frac{1}{\sqrt{2}} V = \frac{\sqrt{2}}{4} V$$

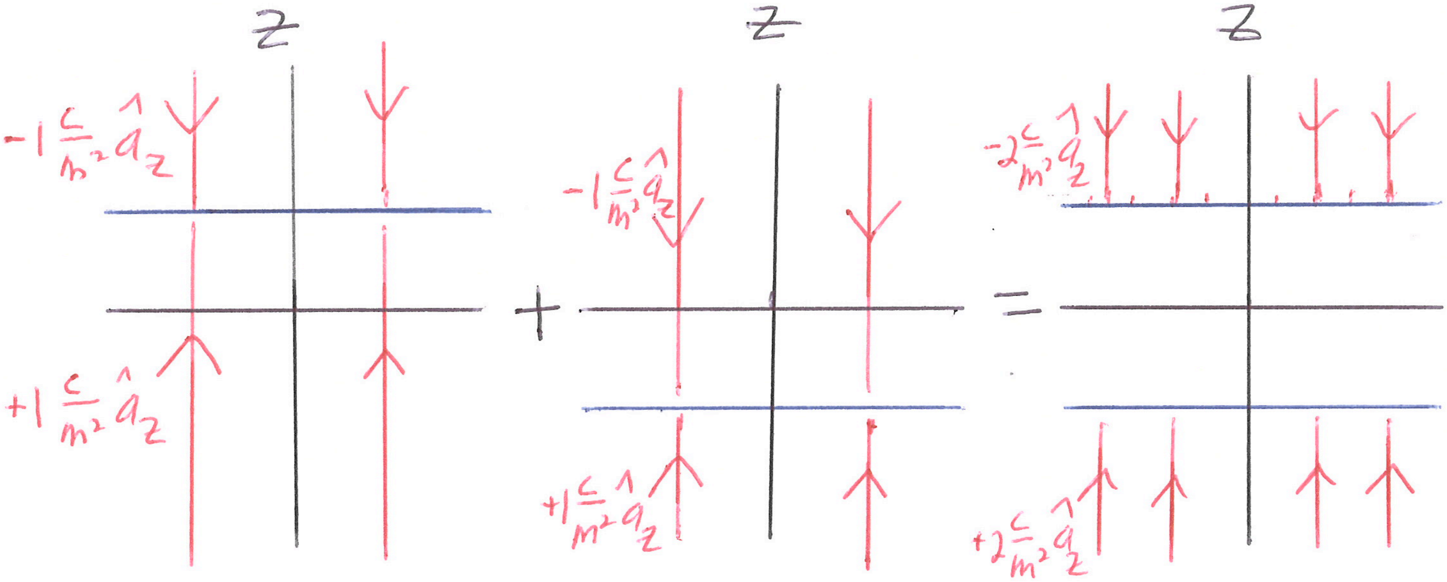
(5 pts) B) what is the voltage at (0, 0, 0)

$$V(0,0,0) = \frac{1}{4\pi\epsilon_0}\frac{\pi\epsilon_0}{1} + \frac{1}{4\pi\epsilon_0}\frac{\pi\epsilon_0}{1}$$

$$V(0,0,0) = \frac{1}{2} V$$

(12 pts) 7. The z = 1 m plane contains a sheet charge density of $-2\,\frac{\mathrm{C}}{\mathrm{m}^2}$ and the z = -1 m plane a sheet charge density of $-2\,\frac{\mathrm{C}}{\mathrm{m}^2}$. Determine the Electric flux density everywhere.

To determine flux density caused by the two planes, we can add the electric flux density caused by each plane as shown,

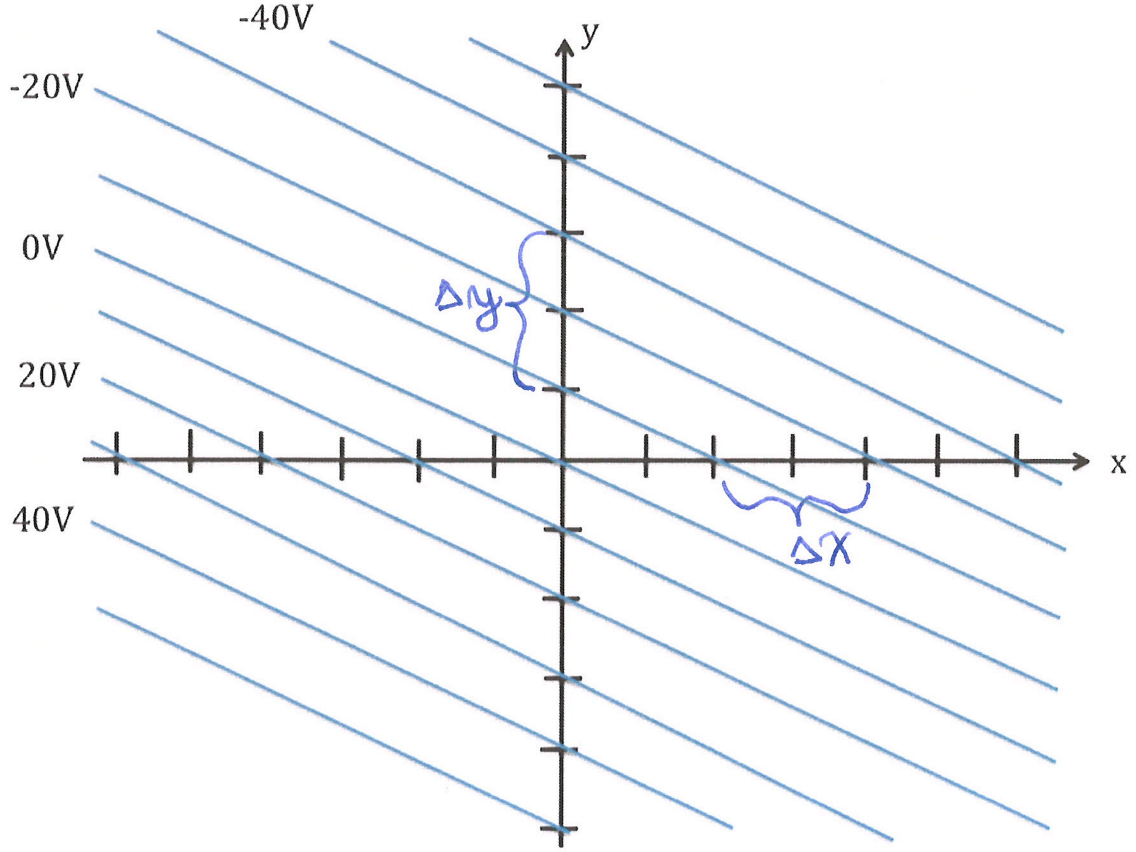


$$\vec{D} = -2\frac{C}{m^2}\hat{a}_z \quad \text{for} \quad z > +1$$

$$= 0 \quad \text{for} \quad +1 > z > -1$$

$$= 2\frac{C}{m^2}\hat{a}_z \quad \text{for} \quad z < -1$$

(10 pts) 8. Shown are infinite planes of equipotential that are parallel to the z-axis. The ticks on the x- and y-axis represent 1 m steps. The potential step between equipotential surfaces is 10V as shown. Determine the electric field intensity.



$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{a}_x - \frac{\partial V}{\partial y}\hat{a}_y - \frac{\partial V}{\partial z}\hat{a}_z$$

$$= -\frac{\Delta V_x}{\Delta x}\hat{a}_x - \frac{\Delta V_y}{\Delta y}\hat{a}_y$$

$$= -\frac{(-20V)-(-10V)}{4m-2m}\hat{a}_x - \frac{(-30V)-(-10V)}{3m-1m}\hat{a}_y$$

$$= \frac{10V}{2m}\hat{a}_x + \frac{20V}{2m}\hat{a}_y$$

$$= 5\frac{V}{m}\hat{a}_x + 10\frac{V}{m}\hat{a}_y$$

(2 pts) 9. A)What are the units of the electric field intensity?

$$\frac{V}{m}$$

(2 pts) 9 B) What are the units of the electric flux density?

$$\frac{C}{m^2}$$

(10 pts) 10. How much energy is stored in an arrangement of two point charges, one of charge $Q_1 = 4\pi\varepsilon_O$ C at location (-1m, 0, 0) and one of charge $Q_2 = 1$ C at (-2m, 0, 0)?

Positioning the $Q_2 = 1C$ charge first requires no work. The work to now position the $Q_1 = 4\pi\epsilon_0$ C charge is,

$$W = Q_1 V_{12} = (4\pi\epsilon_0)\frac{1}{4\pi\epsilon_0}\frac{1}{1} = 1\ J$$

energy stored $= W = 1$ J

(5 pts) 11. Which one of the following statements best describes the equipotential surfaces surrounding a point charge?

A) The equipotential surfaces are planes extending radialy outward from the charge.

B) The equipotential surfaces are curved surfaces surrounding the charge, but only one passes through the charge.

C) The equipotential surfaces are concentric cubes with the charge at the center.

[✓ D)] The equipotential surfaces are concentric spheres with the charge at the center.

E) The equipotential surfaces are concentric cylinders with the charge on the axis at the center.

(10 pts) 12. If the electric field intensity is given by

$$\mathbf{E} = \frac{1}{\rho}\hat{\mathbf{a}}_\phi \ \frac{V}{m} \ \text{for } 0 < \phi \le \frac{\pi}{2}$$

How much work is done moving a $\frac{24}{\pi}$ C charge from $(10, \frac{\pi}{3}, 3)$ to $(2, \frac{\pi}{4}, 1)$?

$$W = -\int Q\vec{E} \cdot \vec{dl}$$

$$W = -\left(\frac{24}{\pi}C\right)\int_{(10,\frac{\pi}{3},3)}^{(2,\frac{\pi}{4},1)} \left(\frac{1}{\rho}\hat{a}_\phi \frac{V}{m}\right) \cdot \left(d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a}_\phi + dz\,\hat{a}_z\right)$$

$$= -\left(\frac{24}{\pi}\right)\int_{\pi/3}^{\pi/4} d\phi \ \ J$$

$$= -\frac{24}{\pi}\,\phi\Big|_{\pi/3}^{\pi/4} \ J = -\frac{24}{\pi}\left(\frac{\pi}{4} - \frac{\pi}{3}\right) J$$

$$= -24\left(\frac{1}{4} - \frac{1}{3}\right) J = -24\left(\frac{3}{12} - \frac{4}{12}\right) J$$

$$= 2\ J$$